

# Assessing Growth and Development of the Facial Profile

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## Abstract

The study of growth and development of the facial profile is of interest to clinicians and researchers in the fields of pediatric dentistry, orthodontics, and craniofacial surgery, enabling diagnosis, planning, and evaluation of treatment. Until recently, craniofacial studies addressed facial growth, facial asymmetry, and gender differences by examining changes in size. However, size changes alone do not represent fully the complicated process of craniofacial growth which also involves changes in shape. The shape of the facial profile can now be quantified with Fourier analysis, contributing to a better understanding of growth. A combination of recently developed methods, such as 3-dimensional facial morphometry and Fourier analysis, should allow a more comprehensive knowledge of growth and development of the craniofacial structures, including the facial profile. This article examines various methods for assessing facial growth and development currently available with particular reference to the facial profile, and addresses the value of Fourier analysis in assessing shape changes. (*Pediatr Dent.* 2003;25:103-108)

**KEYWORDS: FACIAL PROFILE, GROWTH AND DEVELOPMENT, FOURIER ANALYSIS**

*Received April 10, 2002 Revision Accepted September 25, 2002*

Growth and development of the craniofacial structures have been studied extensively, as many clinical disciplines rely on the understanding of these processes for diagnosis, timing, and planning of treatment. This knowledge is important to clinicians and researchers in disciplines such as pediatric dentistry, orthodontics, and craniofacial surgery—enabling detection of normal or abnormal changes, assistance in diagnosis and treatment planning, and prediction of posttreatment outcomes.<sup>1</sup> Several methods have been used to investigate growth and development in relation to changes in both size and shape. Changes in size have been studied by direct measurements (anthropometry),<sup>2</sup> metric analysis of hard and soft tissues (cephalometry and photography),<sup>3-7</sup> and 3-dimensional studies of soft tissue landmarks.<sup>8-13</sup> Recently, changes in shape have been studied by Fourier analysis.<sup>14-20</sup>

One of the important effects of growth and development is change in the facial profile, whereby different components achieve balance through the cellular and tissue control processes of morphogenesis.<sup>1</sup> Growth involves change of the components, whereas development involves the components reaching a state of structural and functional equilibrium.<sup>1</sup> Craniofacial studies have aimed at

determining size and shape changes of facial components including timing, direction, and magnitude of change during growth and development. This paper examines various methods for assessing facial growth and development currently available with particular reference to the facial profile, and addresses the value of Fourier analysis in assessing shape changes.

## Direct assessment of growth using facial anthropometry

Direct measurement of facial landmarks (anthropometry) allows a 3-dimensional study of the soft tissue facial profile. Because measurements are made directly on the subjects and the landmark coordinates cannot be digitized to allow other measurements at a later stage, the approach is time consuming. Inaccuracies may occur in determining landmarks and soft tissues may be deflected by pressure from the measuring device.<sup>10-12,21</sup> Longitudinal studies permit changes to be evaluated during maturation and over time. However, data acquisition is difficult as the study participants may fail to return for recall and few large study populations have been followed over long periods. Consequently, anthropometric studies of facial profiles have been

few and cross-sectional in design. Cross-sectional studies are easier to perform and allow study populations to be investigated with less cost.<sup>2,11</sup> However, in cross-sectional studies, different individuals are examined and the results may be biased.

A cross-sectional study of facial growth was performed at the Craniofacial Measurement Laboratory, University of Toronto, Canada, on 1,594 Caucasian subjects aged 1 to 18 years.<sup>2</sup> Vertical, horizontal, and sagittal measurements were performed with a sliding caliper. Descriptive in nature, the study concluded that, with age, major changes occurred in facial proportions until maturity, resulting from different growth rates of the width, height, and depth of the face. Males had a later maturation age of 15 years in total facial height, width, and mandibular height compared to females, who had a maturation age of 13 years in facial height and width, and in maxillary and mandibular depth. Mandibular width matured earlier than maxillary dimensions in both sexes (males at 13 years, females at 12 years).<sup>2</sup>

### **Cephalometric analysis as a widely used clinical assessment**

Cephalometry is used widely for growth analysis, diagnosis, treatment planning, monitoring of therapy, and evaluation of treatment outcomes.<sup>22</sup> Both hard and soft tissues can be examined. Linear and angular measurements can be compared over time, and radiographs taken at different times under standardized conditions can be superimposed using relatively stable structures. However, cephalometric analysis remains a 2-dimensional representation of 3-dimensional features, resulting in vertical and horizontal displacement of structures in relation to the radiographic film. Facial asymmetry cannot be assessed, and the technique is subject to magnification and distortion of size, positioning, and processing errors, and difficulty in determination of anatomical landmarks.<sup>22,23</sup>

One of the largest longitudinal studies utilizing cephalometry was the Bolton study, conducted at Case Western Reserve University from the 1930s to 1973. Approximately 22,000 recordings of 5,000 individuals of European descent were studied between 1 and 18 years of age.<sup>3</sup> Using determination of landmarks, cephalometric tracings, and measurement of changes, the study established the Bolton standards of dentofacial growth.<sup>3</sup> These annual standards are available for males and females as transparencies consisting of facial landmarks, lines, and angles representing lateral view norms from ages 1 to 18 years and frontal view norms from ages 3 to 19 years.<sup>3</sup> Both soft tissue and skeletal profiles are available, and the clinician may choose a suitable landmark for superimposition of cephalometric radiographs.<sup>1,3</sup> These standards are still widely used for comparative studies of both hard and soft tissue profiles. Due to ethical considerations and possible adverse effects from multiple radiation exposures, annual radiography of nonpatient subjects is now restricted. The cephalometric records in the Bolton study are, therefore, invaluable and have been reexamined in recent craniofacial growth studies.<sup>5,12,14</sup>

### **Gender differences in growth studies**

A recent University of Michigan study of cephalometric radiographs from the Bolton study used linear and angular measurements to evaluate gender differences in normal craniofacial growth.<sup>5</sup> Study of the records of 16 males and 16 females (Caucasian) at ages 6, 9, 12, 14, 16, and 18 years indicated that sexual dimorphism started at 9 years of age and was most apparent at 14 years of age and onwards for most skeletal measurements, which is the time when females reach their final size while males continue to grow.<sup>5</sup>

Soft tissue investigations from cephalometric radiographs have investigated profile changes, comparing measurements of landmarks at different ages. A combined longitudinal and cross-sectional study at the Child Research Council in Denver, using cephalometric radiographs of 17 males and 23 females at the ages of 7 to 8 years and 17 to 18 years, indicated sexual dimorphism in the nose, lips, and chin.<sup>4</sup> Males showed a larger increase in size of these structures and this extended over a longer time period than in females.<sup>4</sup> Adult female size was achieved in most measurements by 15 years of age, whereas males continued to show linear increase until the final measurements at 18 years. The proportions of adult size attained in different facial structures at 5, 15, and 18 years of age in males and females have been tabulated, and such data can assist in planning treatment, anticipating growth-related changes, and predicting posttreatment outcomes.<sup>4</sup>

Confirmation of gender differences in the soft tissue profile was found in a combined longitudinal and cross-sectional study of cephalometric radiographs in Nijmegen, Netherlands.<sup>6</sup> A total of 82 subjects (45 females, 37 males) was studied longitudinally from ages 9 to 22 years. Gender-different growth patterns commenced at 9 years, when the soft tissue structures of girls changed in size rapidly compared with boys who were still growing slowly, to reach a similar rate of growth at 12 years of age when the velocity curves overlapped. After this age, the velocity curve decreased in girls and increased in boys until final measurement of the soft tissue profiles.<sup>6</sup>

A cephalometric study of soft tissue profiles conducted by the Iowa Facial Growth Study used longitudinal records of 20 females and 15 males aged 5 to 25 years.<sup>7</sup> Similar direction and magnitude of changes occurred in males and females, but the greatest changes in soft tissue profile occurred earlier in females (10 to 15 years) than in males (15 to 25 years).<sup>7</sup> In both sexes, there was an increase in total facial convexity from 5 years to adulthood due to a greater increase of the nasal prominence relative to the remaining soft tissue profile. Late changes were found in both the upper and lower lips, as they became more retruded in position, even up to 45 years of age.<sup>7</sup>

### **Three-dimensional methods to assess craniofacial growth**

As indicated above, anthropometric studies are limited in accuracy, and cephalometric radiographs and photographs

provide only 2-dimensional representations of growth. Since growth is a 3-dimensional process, it is argued that all directions of growth should be assessed. Three-dimensional approaches to study growth now include stereophotogrammetry and 3-dimensional facial morphometry.<sup>8,9,13</sup> These techniques are useful in assessing facial volume and facial asymmetry, are noninvasive and relatively inexpensive, and are applicable to longitudinal studies of large numbers of subjects. However, the techniques are limited to recording coordinates of cutaneous points and information may be lost during reconstruction of the face in a coordinate system (the process of "facial extraction").<sup>10</sup>

Stereophotogrammetry uses a dual-purpose stereometric camera to stereoscopically record a pair of facial photographs and a contour plotting device; the result is a computerized facial map.<sup>8,9</sup> Anatomical facial landmarks are plotted in 2 mm intervals from the tip of the nose to each landmark, and presented in x, y, and z coordinates. The method allows linear measurements, which can be combined to measure volume.<sup>9</sup> The technique has been used in longitudinal studies at the University of Sheffield, England, on 26 boys and 26 girls to determine the adolescent growth spurt of facial soft tissues, including the nose.<sup>8,9</sup>

Measuring several facial parameters, Burke and colleagues (1988) found the adolescent growth spurt of the facial soft tissue coincided in timing with general somatic growth, although some variations were noted.<sup>8</sup> Boys experienced this spurt at 12 to 13 years of age; girls varied in the observed parameters, although a spurt was seen at 12 to 14 years old. Concerning the nose, an adolescent growth spurt was noted with a greater manifestation in anterior nasal growth compared to nasal height and width, which were influenced more by overall facial growth.<sup>9</sup> This spurt occurred at 13 to 14 years of age in boys, and at 9 to 10 years of age in girls.<sup>9</sup>

A 3-dimensional method for craniofacial growth studies, 3-dimensional facial morphometry (3DFM), utilizes an ELITE television image analyzer (ELaboratore di Immagini Televisive, BTS, Milan, Italy), which consists of 2 charge coupled device cameras to record the image, hardware for identification of soft tissue landmarks, and software for reconstruction of x, y, and z coordinates of the landmarks.<sup>10</sup> After marking the facial landmarks visually, the subject is positioned in front of the cameras to record the image from different angles. The landmarks are then translated by the ELITE system and the face is reconstructed with the x, y, and z coordinates.<sup>10</sup>

A series of studies has been performed with 3DFM at the Università degli Studi di Milano, Italy, involving 2,023 examinations.<sup>11-13</sup> The studies were both longitudinal and cross-sectional in design, examining 22 facial landmarks on 1,347 subjects (northern Italians) aged 6 to 32 years.<sup>11-13</sup> Facial volume changes over time were computed and compared for males and females. Growth patterns were similar for both genders until age 11 years, then differed significantly thereafter.<sup>12</sup> A growth spurt in facial volume was evident in females by 11 to 12 years then growth declined rapidly and ceased by

14 to 15 years. In males, the growth spurt was evident by 11 to 12 years and continued at a similar rate to 16 to 17 years.<sup>12</sup> Facial volumes were always larger in males than females in all age groups, but were similar during the youngest period (6-7 years) and preadolescence (11-12 years).<sup>12</sup> Linear facial measurements in males were wider, longer, and deeper than in females of the same age group; the differences were statistically significant in all age groups, and especially after 14 years of age.<sup>11</sup> These findings confirmed those of earlier soft-tissue cephalometric studies.<sup>2,4,24,25</sup> The areas of the face most influenced by sexual maturity were the nose and lips; features of particular interest to clinicians. In the future, this 3-dimensional approach could be combined with cephalometry for diagnosis and treatment planning.<sup>13</sup>

### Limitations of metric measurements

The preceding approaches assess growth using conventional measurements of linear distances, angles, and ratios—techniques that were developed originally for measuring regular geometric objects. Despite ease of use, metric analysis may not be appropriate for measurement of irregular and complex biological forms such as the face.<sup>19,20</sup> Shortcomings include the limited number and wide spacing of landmarks used to assess complex forms, bias and subjectivity in choosing landmarks to represent form, and difficulties in standardizing for size.<sup>26-29</sup> The complex structures of the face cannot be represented fully by combining separate measurements of the different facial parts.<sup>1</sup> Further, study of the process of growth and development in young subjects is influenced by size differences which mask the more subtle shape differences.<sup>1,18</sup> In summary, metric measurements are sufficient to evaluate dimensions of craniofacial components and size, but are inadequate to quantify shape and changes in shape that occur with growth and development.<sup>13,17,18</sup>

### Fourier analysis for assessing changes in shape

A mathematical approach to quantify shape in biological forms has been developed in the form of a Fourier analysis (FA). The analysis was first described by Jean Baptiste Joseph Fourier (1768-1830), and it is a development from pure mathematics now applied in fields as diverse as physics, astronomy, optics, and electrodynamics, and, more recently, in the fields of pattern recognition, biology, and medicine.<sup>19</sup>

Fourier analysis is a curve-fitting procedure representing boundaries that address the outline of objects. It is based on the separation of complex waveforms with a mathematical function to form a series of sinusoidal waves, or harmonics, of different frequencies.<sup>27</sup> This enables a mathematical description of the outline of an object, quantitative analysis of global shape characteristics, and comparison of outlines of different objects. The analysis is conducted on scanned frontal and profile photographs taken simultaneously using an orthogonal camera system. The facial features are then extracted digitally, and shapes are computed using FA and Fourier descriptor software. Changes

in the outline of an object over time can be compared without the influence of size, spatial orientation, or relationship to reference planes.<sup>13,14,16,18</sup> Based on these advantages, the analysis has been proposed to assess shape changes in growth studies.<sup>15,17,18,20</sup>

The value of FA in representing the facial profile has been described. A study of both hard and soft tissues of the facial profile (from the nasal bridge to the chin) was conducted on 77 males aged 25 to 74 years in the VA Dental Longitudinal Study in Boston.<sup>30</sup> Twenty harmonics were used to assess the fit of the computed function with the observed profile outline, resulting in an overall excellent fit with less error than manual tracing and fewer digitizing errors.<sup>30</sup> The FA has also been used to assess age differences in the Bolton tracings.<sup>14</sup> Seven cephalometric landmarks were connected and used to calculate the centroid (a neutral center within a represented object that remains constant with rotation, similar to a center of gravity). Using 20 harmonics, an excellent correlation was found between the mathematical reconstruction and the original plot; a lesser value was found when 6 harmonics were used. However, this study was still dependent on landmark identification, and the investigation was performed on the outline created from connected landmarks instead of the facial profile *per se*.<sup>14</sup>

Other investigations with FA have assessed facial growth. In a cross-sectional study at the Università degli Studi di Milano, Italy, of 122 subjects (northern Italian) aged 7 to 15 years, seven cephalometric landmarks were connected and superimposed.<sup>17</sup> Shape was found to be influenced by gender in all age groups, except for 10 to 12 year olds. Boys showed greater variations of shape in different age groups than girls.<sup>17</sup> Soft tissue facial profile growth was also investigated at the same institution, using the Bolton standards and FA.<sup>18</sup> Soft tissue landmarks were traced, plotted as polar coordinates, and compared between the ages of 1 to 18 years. The study concluded that facial soft tissue size and shape were significantly determined by age and that soft and hard tissue changes were not correlated linearly and, therefore, should be assessed independently.<sup>18</sup> The growth of facial soft tissues was evaluated further in a cross-sectional study at the same institution, studying 144 children in 2 age groups—6 to 7 years old and 9 to 10 years old—as part of a 3DFM study that provided frontal and lateral soft tissue facial landmarks.<sup>15</sup> Using FA, the landmarks were plotted as coordinates. Shape modification during growth was again found to be gender-specific, with boys and girls showing different timing and magnitude of change. Shape differences were most evident in both genders in the lower one-third of the facial profile, expressed more in a vertical direction than a transverse direction. Facial asymmetry was found to increase between the ages of 6 to 7 years and 9 to 10 years, particularly in girls.<sup>15</sup> Conventional FA is relatively straightforward, and the coefficients or harmonics can be related to biological meaning.<sup>31</sup> However, the use of FA is limited to relatively simple morphological forms, and it is difficult to fit into complex or irregular forms.<sup>19,28</sup>

## Recent developments in elliptical Fourier analysis

To overcome the problems associated with conventional FA, an elliptical Fourier analysis (EFA) was developed recently to investigate more complex morphological forms. Originally developed in 1982 by Kuhl and Giardina,<sup>32</sup> the outcome is a set of numbers (harmonics or coefficients), selected based upon the detail required. Ellipses are produced when the separate harmonics are plotted and, on summing these, combine into the observed form. The first few harmonics or lower order represent the global features of shape, while the higher order represent the facial profile in more detail.<sup>28</sup> The number of harmonics used depends on the amount of detail required; 30 or fewer harmonics can represent the facial profile accurately.<sup>33</sup> The advantages of EFA include the ability to define complex structures the relative independence from the centroid and landmarks, and the consistent orientation of structures that allow comparison of different objects.<sup>19,34</sup> However, interpretations may be difficult, since each harmonic is an elliptical shape.<sup>14,27</sup>

With reference to growth, EFA has been used to calculate a “morphological distance” (MD) to measure differences in shape.<sup>13,16,34</sup> The MD is the distance between each harmonic pair of 2 different objects with a given mathematical function. For example, if the objects compared are identical, the MD would be 0. The method allows comparison of different objects or the same object at different times. Differences in the Bolton standards of dentofacial growth have been examined using EFA, and it was found that, between ages 1 to 17 years, the greatest difference in shape occurred during the first year, and, thereafter, the differences progressively declined, with minimal differences occurring after 15 years of age.<sup>16</sup>

A longitudinal study at the University of Glasgow Dental School, Scotland, used cephalometric radiographs of the mandible from Leighton’s archival growth study<sup>35</sup> of 24 subjects 9, 11, 13, and 15 years of age, and described shape changes with EFA.<sup>20</sup> Mandibular points were traced on the outline of the mandible, and the distances of these points from the centroid were calculated. Mandible outlines were also superimposed to visually inspect shape changes. No gender differences were found; however, significant shape changes were noted from 11 years onwards involving the mandibular incisor area, mental region, body of the mandible, and the gonial angle.<sup>20</sup> Small sample size can limit such studies, and the possibility of tracing and point identification errors (especially in overlapped areas), and errors incurred in fitting the reconstructed outlines in areas of dramatic contour change (eg, tip of the incisor and tip of the coronoid process), are possible.

## Twin studies using elliptical Fourier analysis

Knowledge of genetic and environmental influences on the craniofacial structures during growth requires more attention.

Better understanding of hereditary factors would assist clinicians in treatment planning, enabling prediction of craniofacial areas more susceptible to treatment or prone to relapse following treatment. Recent craniofacial growth studies have used twins as a tool to determine genetic and environmental factors.<sup>36,37</sup> In twin studies, identical twins (monozygotic twins) are seen as genetically identical, and, thus, differences among them are due to environmental influences, whereas nonidentical twins (dizygotic twins) share similar environmental experiences as identical twins, but share the same genes as siblings.<sup>38</sup> Previous growth studies in twins have utilized cephalometric radiographs only.<sup>36,37</sup>

Craniofacial twin studies have been performed with EFA, resulting in quantification of the facial profile of twins and relatively good classification of twins.<sup>39,40</sup> A study of genetic and environmental influences on the facial profile using EFA was conducted at the University of Melbourne, Australia, on 79 twin pairs (37 identical, 42 nonidentical) aged 4 to 6 years old.<sup>40</sup> This study found that EFA could classify twin type, quantify the facial profile of the twins, and differentiate some details, but was not able to specify which structures were genetically or environmentally influenced at this early age.<sup>40</sup> Follow-up studies are in progress using EFA and a longitudinal sample of the same twins, assessing shape changes, and identifying genetic and environmental contributions during the processes of growth and development.<sup>41</sup>

### Future possibilities in the study of craniofacial growth

To date, craniofacial studies have attempted to determine growth changes, signs of facial asymmetry, gender differences, and development of facial maturity by conventional metric approaches to changes in size and by measuring linear distances, angles, and ratios. Anthropometry, cephalometry, and 3-dimensional analysis are some examples of this approach. Three-dimensional imaging has been expanded recently using laser-scanning techniques.<sup>42</sup> Determination of changes in shape is now possible with conventional and elliptical Fourier analyses. Future craniofacial studies should examine both size and shape changes during growth and development. In particular, the application of Fourier analysis in determining shape changes during development of the human face needs further investigation. Longitudinal studies combining Fourier analysis with 3-dimensional facial morphometry could provide valuable information on craniofacial growth from different angles and views and for different populations including children with craniofacial abnormalities. Such studies could be used to establish standards or norms, which could then be made available to clinicians for comparisons with individual subjects.

### Conclusions

In general, it can be concluded that growth patterns of the face are similar in males and females in the young age stage. Gender-different growth patterns can then be detected by 9 years of age, and, thereafter, are most apparent until females cease

growing but males continue to grow. This age of maturity varies—in females it is reported to be 13 to 15 years old and in males 17 to 25 years old. Late changes are observed to continue occurring in both the upper and lower lips, even into adult years, resulting in continued change in the facial profile.

While metric measurements are sufficient to assess dimensions of craniofacial components and size, they are inadequate to quantify shape and changes in shape that occur with growth and development. Better understanding of genetic and environmental influences during the process of growth and development of craniofacial structures is also required. Advances in the knowledge of craniofacial growth should include both size and shape changes in all dimensions. A combination of recently developed methods applicable to longitudinal studies, such as 3-dimensional facial morphometry and Fourier analysis, should allow a more comprehensive knowledge on growth and development of the craniofacial structures, allowing improved prediction of clinical outcomes.

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